The relation between tympanic membrane higher order modes and standing waves

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Abstract. Here we address the question of the specific relation between Tympanic Membrane (TM) higher order modes, and the existence of standing waves. These questions relate to the nature of the middle ear (ME) as a cascade of transmission lines, and what happens when the matching goes from nearly perfect, when the normal cochlear is the TM load, to totally out of balance, due to either ME or cochlear pathology. This is important for the diagnoses of a pathology given the ME reflectance.

INTRODUCTION

Two related middle ear models of Parent and Allen [14, 15] (aka, PA07) have characterized normal and pathological middle ear acoustic impedance and forward pressure being delivered to the cochlea. Thus it was against such data that the model was compared. The model was not tested for reverse traveling waves, such as acoustic emissions coming from the cochlea. Since this application is of interest, it seems that PA07 needs to be extended and tested to account for reverse transmission data. Presently such data is limited. Furthermore reverse transaction is likely more sensitive to complex wave propagation on the TM due to higher order modes (HOM). This topic will be explored further here.

Since the PA07 model was first introduced, new experimental results have come to light that also need to be discussed. For example, it has been observed in several laboratories that there are complex modes on the TM [3, 5, 22, 23], consistent with TM standing waves. Here we shall discuss these new observations re PA07.

All of these observations beg the question as to the true nature of wave propagation on the TM [7, 8]: “Is the TM best modeled as a 1, 2 or 3D structure,” and “Are the waves best modeled by a horn (wave equation) or shell?” What are the appropriate boundary conditions? It is frequently observed, e.g., [7], that finite element models are so complex that they cannot easily be properly formulated, and/or interpreted, until these more basic questions have been addressed.

MODEL BASICS

The theory of the PA07 model [14, 15] is from the class of of digital wave guides [4], a field initiated by a Kelly and Lochbaum [10]. We use the following building blocks: First is the application of *Gauss’s Law*: the acoustic plane-wave input velocity is split into 143 lumens perpendicular to the plane-wave iso-pressure surface, in proportion to the area of each lumen. Next the wave velocity of each transmitted plane-wave lumen v_n^+ is propagated in the canal with a lumen-dependent delay τ_n^C, and there reflected by a reflection coefficient determined by the change in the specific impedance between air the the TM.

The transmitted TM wave component u_n^+ is further delayed by an amount τ_n^{TM}, due to the TM’s much slower propagation wave speed, where the inward traveling wave is summed at the manubrium. A corresponding outward traveling TM component u_n^- is small, relative to in inward bound component u_n^+, as it is either radiated back into the ear canal (v_n^-) or perfectly absorbed at the tympanic ring (TR) boundary condition (BC) (i.e., $U_{\pm N}^- = 0$). This last assumption seems inconsistent with recent observations of standing waves on the TM. Thus it seems wise to revisit this TR boundary condition (TR-BC).

Putting this all together we have (Fig. 1)

$$\begin{bmatrix} u_n^+ \\ v_n^- \end{bmatrix} = \begin{bmatrix} -R_n e^{-j\omega 2\tau_n^{TM}} & (1-R_n)e^{-j\omega(\tau_n^C+\tau_n^{TM})} \\ (1+R_n)e^{-j\omega(\tau_n^C+\tau_n^{TM})} & R_n e^{-j\omega 2\tau_n^C} \end{bmatrix} \begin{bmatrix} u_n^- \\ v_n^+ \end{bmatrix}. \tag{1}$$

On the left are the output wave variables $\left[u_n^+, v_n^-\right]$ indexed by the lumen index n, while on the right are the input wave variables $\left[u_n^-, v_n^+\right]$. For example, when the manubrium input velocity $U_m^- \equiv \sum_n u_n^-$ is zero, $u_n^- \equiv U_m^- A_n/A_n' = 0$.

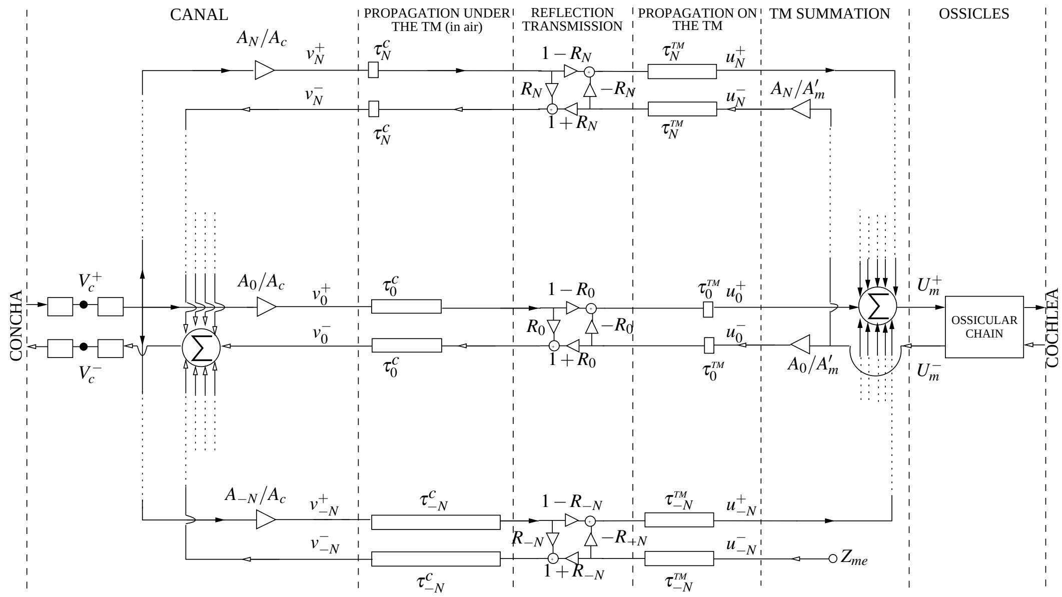

FIGURE 1. This figure is a revised Fig. 4 from Parent and Allen [14], with improved variable labeling.

Likewise when the canal input velocity $V_c^+ \equiv v_n^+$ is zero, $v_n^+ \equiv V_c^+ A_n/A_n = 0$. These represent input boundary constraints on the canal and cochlear inputs from the two ends.

In the physical middle ear near the TM, large local reactive fields in the form of *higher order modes* (HOMs) can be present, due to complex non-planer wave motion on the TM. This is most obvious for reverse traveling waves radiated from the TM, driven in a highly asymmetric manner by the rotations of the manubrium, which is only attached to one half of the TM.

However, HOMs in the canal cannot propagate more than a few [mm] for frequencies f less than the lowest *modal cutoff frequency* f_c, (i.e., $f \ll f_c \approx 27 < c/d$ [kHz]). This condition strictly holds when the wavelength $\lambda = c/f$ is much greater than the human ear canal diameter $d \approx 7.5$ [mm] (i.e., $\lambda \gg d$). The common way of stating this long wavelength condition is $kd < 1$, where $k = 2\pi/\lambda$ is the wave-number [2, 13, 16]. Thus complex canal modes, near the TM, rapidly degenerate into a plane wave within this distance as they travel into the canal.

For a circular tube of diameter $d = 2a = 0.75$ [cm], the lowest modal cutoff frequency f_c, where $k_c = 2\pi f_c/c$, is determined by the smallest root $r_1 = k_c a = 1.84$ of the Bessel function, namely the arcBessel of $J_1(r_n) = 0$, with r_n being the roots [12, 13]. This lowest cutoff frequency, $f_c = 1.84c/\pi d \approx 27$ [kHz], is defined as the frequency where the wave number goes from imaginary to real [13]. When $f < f_c$ the wave decays as

$$e^{-2L_1\omega_c\sqrt{1-(f/f_c)^2}/c} \approx e^{-4\pi L_1 f_c/c}.$$

For example, for $f = 10 < f_c = 27$ [kHz], at a mid-canal distance of $L_1 = 1$ [cm] from the TM (the ear canal is ≈ 2.5 [cm] long), this would give an attenuation of this lowest HOM by about -87 [dB] (i.e. the wave number jk is real, negative, and large) [13, 24, p. 374]).

Relation between HOMs and TM standing waves

Related to these canal HOM are the observations of TM standing waves, frequently observed in TM laser scanning measurements [5, 22, 23]. This relation comes about because TM standing waves drive HOMs that propagate into the canal in a highly attenuated manner, as stated above. Below 20 [kHz], every such mode is attenuated by more than 80 [dB] 1 [cm] from the TM. Nor can they propagate back through the ossicles, due to their low-pass filtering properties.

The role of HOMs is carefully explained by Karal [9] for the special case of a tube having a discontinuous area. Karal shows how such a junction may be modeled as a series impedance determined by the HOM [9, Eq. 4, Fig. 2]. For plane waves well below cutoff, Karal's equivalent series impedance is a *spreading mass* due to the spreading (or compressing) of the wave as it propagates into the second tube. The magnitude of this mass depends on the ratio of the two radii. Physically this mass accounts for the increase in a frequency-dependent effective length, due to the area change of the second tube [13, Eq. 23.1, p. 234].

For the case of the TM, where the appropriate canal boundary condition is due to the change in the specific characteristic impedance, HOMs still play a role, but must be accounted for by slow wave propagation on the TM, as assumed by PA07 and motivated by Puria and Allen [17].

The retrograde wave propagation is not the same as the forward case, due to the HOMs, caused by the asymmetrical manubrium excitation of the TM. This is an interesting and important issue, related but beyond the present goals.

Finally there is the important open question as to the boundary condition at the tympanic ring (TR-BC). It was tacitly assumed by PA07 that this BC was absorbing, leading to no standing waves on the TM. Again, this assumption was made for simplicity, since the goal was to model the forward acoustics, not reverse acoustics.

TM STANDING WAVES DEPEND ON THE TM-BC

Not discussed in PA07 was the assumption of an absorbing boundary condition at the tympanic ring (the outer boundary of the TM). This condition requires setting $R_{\pm N} = 0$, to absorb any remnant wave volume velocity traveling out from the manubrium that has not been either radiated back into the canal or reflected back toward the ossicles, as defined in Eq. 1 and Fig. 1. The PA07 assumption of the absorbing TM-BC requires that $V_c^- = U_m^-$ which in turn requires that the two "rim" reflection coefficients $R_{\pm N} = 0$. This assumed absorbing TM-BC of PA07 removes the possibility of TM standing waves. Thus the PA07 model would need to be modified to allow such reflected waves, which for pathological ears, could be significant. It would be helpful to revisit this question in the future to account for anomalous responses seen in real data [21]. The TM-BC provides an important extra degree of freedom, required to match experimental data.

In the physical cochlea, the TR-BC will never be ideally matched. Standing waves on the TM are not desirable, as they lead to fluctuations in the forward transfer function. The nature of the TR-BC match will, of course, depend on any pathology, leading to standing waves on the TM, as have been observed. Since PA07 assumed an ideal TM-BC absorbency, they did not observe (since they did not model) TM standing waves. No attention was given to this important issue in PA07.

For the non-ideal case where the absorbing TR-BC does not hold, there must be standing waves on the TM, since some of the outbound TM wave (i.e., u_n^-) will be reflected back by the outer boundary condition $R_{\pm N} \neq 0$. An interesting experimental measure might be to vary the impedance/reflectance stapes boundary condition and measure the ear canal pressure, with the the canal terminated in its characteristic impedance ($\Gamma_c = 0$).

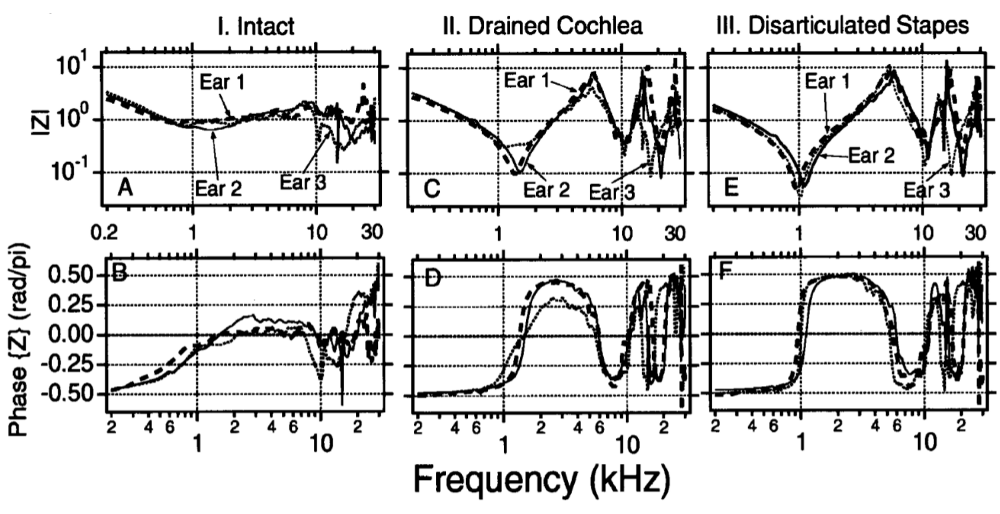

FIGURE 2. This is Fig. 1 reproduced from Puria and Allen [17] showing the input impedance as measured from three different cats, for the same three conditions. I. The intact animal with normal cochlea and normal hearing. II. The cochlea has been drained so that the cochlear fluids are no longer coupled to the stapes footplate. III. The Incudo-Stapidial joint has been cut, mechanically removing the annular ligament, which holds the stapes footplate in the oval window. These experiments were performed by the author at Columbia University around 1982. The data from Ear 1 is the same as that published in [1], where more than these three conditions are shown.

Standing waves on the TM

To study the TM standing wave problem there are a number of basic issues that need to be addressed, and several basic concepts need to be established [17]. The first issue is the variability and the quality of the data across preparations. Understanding this issue requires insight into the nature of the physical preparations. Second, quantifying the standing wave modal frequencies is essential if we wish to model the phenomena. Once one quantifies the system eigen-frequencies, the question then becomes, “What is the best method for measuring the standing wave frequencies?” We need to agree on some physical principles as to where these standing waves come from. Given that, “What is the source of damping of the modes?" and “How do we experimentally control their damping?”.

The first widely acknowledged observations of standing waves was by Khanna and Tonndorf [11]. These early measurements were, as they acknowledged, seriously impacted by the preparation, which was on an excised temporal bone.

The fundamental role of the ME is to deliver the acoustic signal from the canal (air) to the cochlea (water). The middle ear is a cascade of matched transmission lines. Cascaded transmission lines trades mass for delay, with no known negative side effects. For example, by adding joints between the ossicles, which act as shunt compliances, the ossicles mass is reduced at the expense of a small delay [14, 17].

Thus when the cochlea is removed as the load to the ME, standing waves result, because the middle ear transmission line is no longer matched (e.g., Fig. 2). The inherent delay within a transmission line, along with the stapes and TR boundary conditions, determines the system’s eigen-frequencies. Given this understanding, key information can be obtained from the magnitude and phase of the input impedance, or better, its reflectance. The eigen-frequencies f_n are defined by $\angle\Gamma_c(f_n) = n\pi$, conditioned on $\Gamma_s = -1$ (Fig. 2).

A clear demonstration of this principle, in three different living cat cochlea, was first shown by [1] as suggested by Rabbitt and Holmes [19] and subsequently modeled by Puria and Allen [17], which showed for the first time the existence of large delays in the TM, reproduced here as Fig. 2 [17, Fig. 1].

The impedance of the cat ear canal was measured in a normal preparation. Then the cochlea was drained and the canal impedance was remeasured. Further modifications were made, such as cutting the IS joint. This allowed one to directly infer, in living animals, the role of the cochlear load on the input impedance of the ear canal [1].

The results were dramatic, leading to at least five major conclusions:

1. The acoustic input impedance is dominated by the resistive cochlear load. When the cochlea is drained, the input resistance dropped by an order of magnitude. This clearly demonstrates that the cochlear load is resistive and closely matched to the middle ear.
2. The magnitude of the standing wave ratio (SWR) is ≈ 40 [dB]. This is a recurrent issue with ME modeling. Such a large SWR is a direct indication that the normal cat ME has very small loss.
3. The first standing wave zero was between 1-2 kHz, and the first pole around 6 kHz, indicating a very large delay in the middle ear [17].
4. One may conclude that the middle ear is a system of matched transmission lines.
5. The results of the three independent animal experiments are almost identical.

These observation were explored by [17, Fig. 1], who were among the first to suggest that this delay is mostly in the TM (see also Farmer-Fedor and Rabbitt [6], Rabbitt [18], Rabbitt and Holmes [19, 20]).

The above points seem to have been overlooked in reviews and models of ME properties. As an example, Fay et al. [7] conclude that a large number of random modes would explain the tight middle ear coupling. A large number of modes would require a very large delay. This view is contrary to that of a system of matched transmission lines having the observed delay. The three cat ears are nearly identical (in conflict with the “discordent eardrum”proposal of [7]), with a constant real impedance above 0.8 [kHz], and a deep zero at 1 kHz for the dis-articulated stapes case. These three ears with well defined standing waves, are clearly not discordent.

DISCUSSION

The intent of PA07 was to characterize the canal impedance and reflectance, and to provide some estimates of the forward pressure reaching the cochlea. Today there is a need to characterize reverse transmission and understand the role of standing waves on the TM. Achieving these goals will require significant extensions of PA07.

If the results of Allen [1] are typical of other ears, one must conclude that all of the experiments that show TM standing waves are likely impacted by this very sensitive condition of the matched cochlear load. Specifically, when the stapes load (the cochlea) is not normal, TM standing waves must follow. When the ME is matched by a normal cochlear load, the TM standing waves are damped out of existence by a matched cochlear radiation resistance. The degree of such radiative damping depends critically on the precision of the impedance match.

Given the many reports of standing waves, one must ask "How well the cochlear load was controlled in each of these cases?" This sensitivity to cochlear load seems unappreciated, since there must be a delicate balance between the TR-BC and the acoustic load, both being critical factors.

For the case of an acoustic source in the cochlea (i.e., DPOAE reverse transaction) these issues are further complicated by the canal acoustic load, which is rarely specified. The canal input impedance measurement assumes a blocked (zero volume velocity) condition in the ear canal (The impedance is defined as the pressure response to a volume-velocity source, which has a source impedance of zero). When the reverse transfer function is measured, the canal load impedance becomes critical. To know this one needs to know the acoustic source Thevenin impedance. This is rarely measured. In the case of Allen [1], Rochefaucauld and Olson [22], it is known to be matched, due to the use of the "Sokolich transducer," which was carefully designed to be impedance matched [1].

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